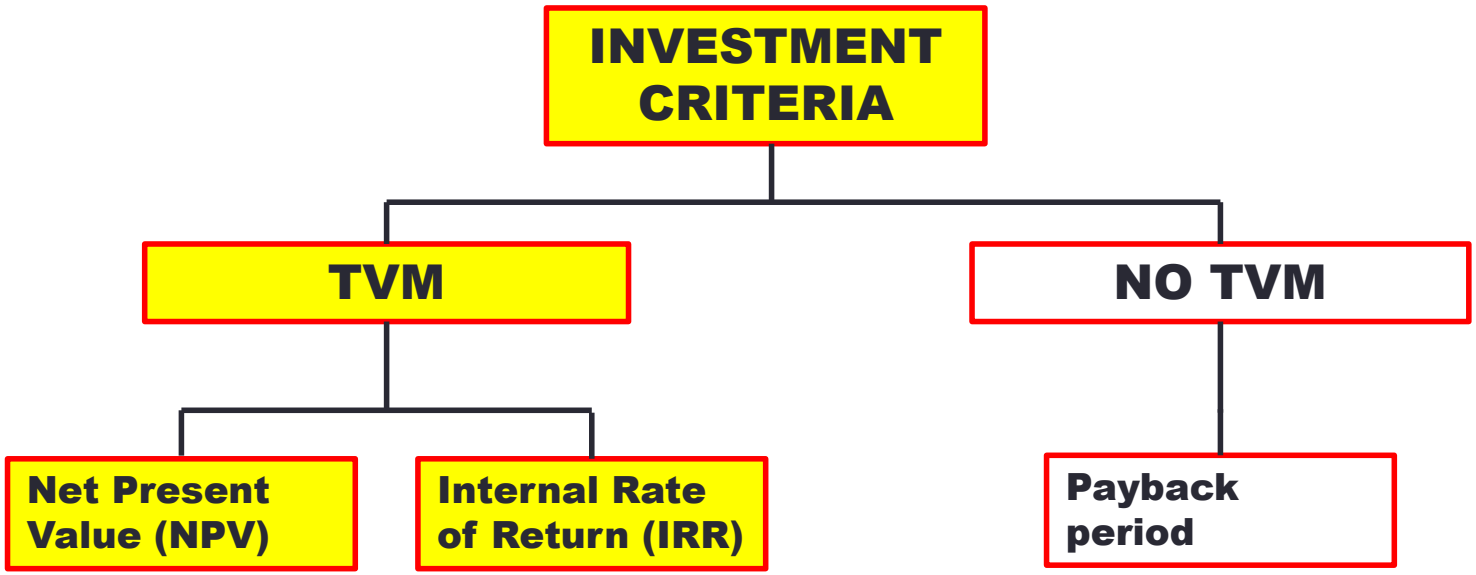
INVESTMENT DECISION RULES

Lecture Outline



Net Present Value (NPV)

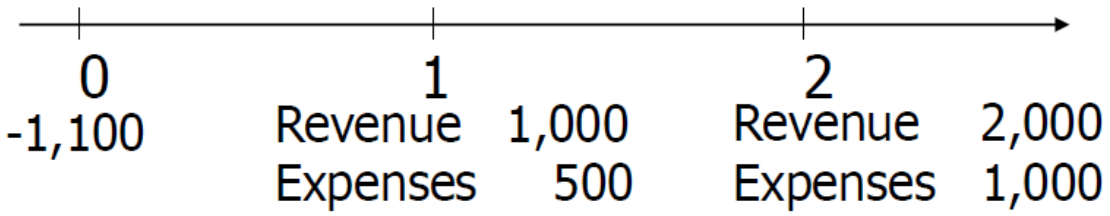
- The difference between the present value of cash inflows of a project and its cost
- How much value is created from undertaking an investment?
  - Step 1: estimate the expected future cash flows (leave it to the chapter of "capital budgeting decision")
  - Step 2: estimate the required return for projects of this risk level (leave it to the chapter of "equity pricing and stock returns").
  - Step 3: find the present value of the cash flows and subtract the initial investment/cost

Net Present Value – Decision Rule

- **If the NPV is positive, accept the project**
  - A positive NPV means that the project is expected to add value to the firm and will therefore increase the wealth of the owners.
  - Similarly, taking an NPV < 0 project will destroy value.

Net Present Value – Example 1

Project X needs initial outlay 1,100, and your required return is 10%. Annual cash revenues and expenses are as follows:



$$NPV = -1100 + \frac{1000 - 500}{(1 + 10\%)} + \frac{2000 - 1000}{(1 + 10\%)^2} = 180.99$$

NPV is positive, accept the project

NPV - Decision Criteria

NPV is the dominant method for making investment decisions

- account for the time value of money
- account for the risk of the cash flows
- provide an indication about the increase in value

Internal Rate of Return (IRR)

This is the most important alternative to NPV

- It is often **used in practice and is intuitively appealing**
- It is based entirely on the estimated cash flows and is ***independent of interest rates*** found elsewhere
- Very similar to finding the ‘rate’ in a multiple cash flow scenario.

Internal Rate of Return (IRR)

- What is the IRR? The discount rate that sets NPV =0

IRR solves:

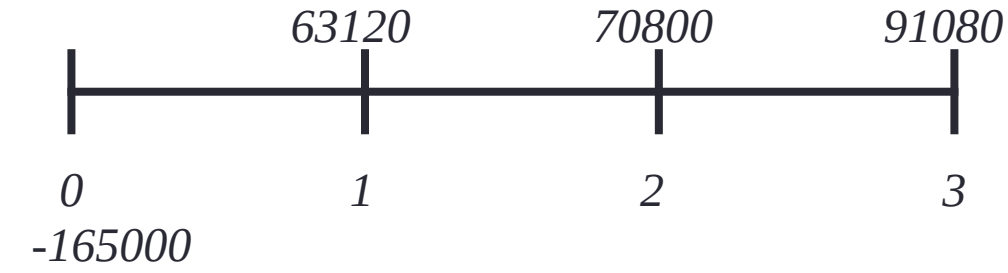
$$\sum_{t=0}^{T} \frac{C_t}{(1+IRR)^t} = 0$$

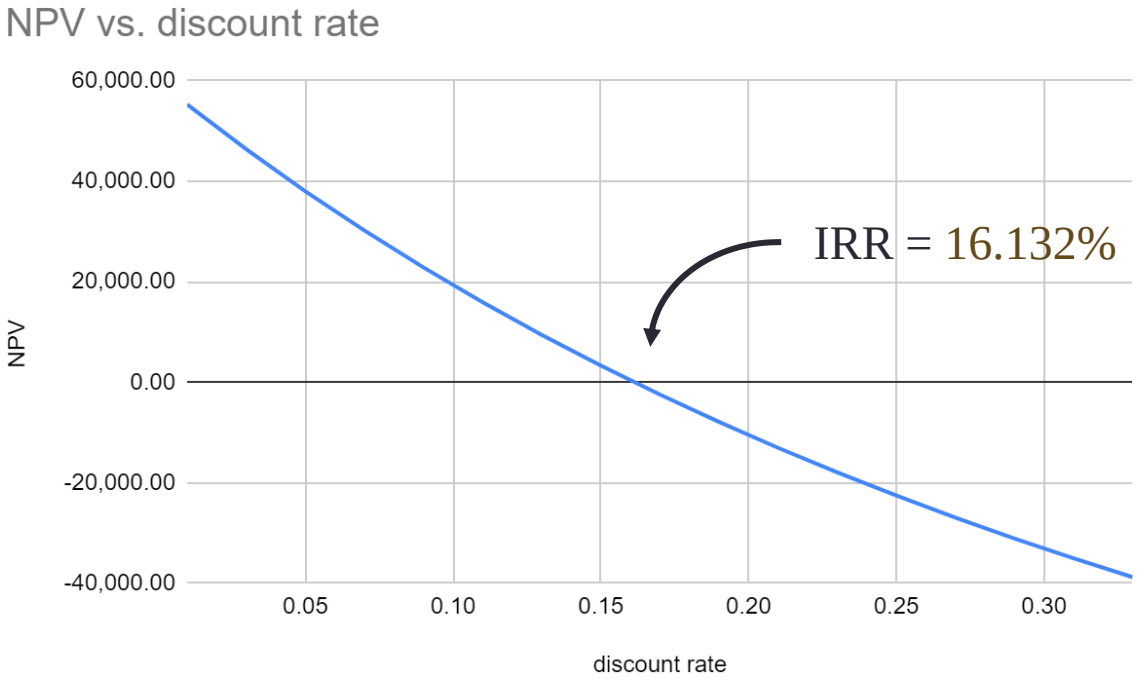
- What is the discount rate that make the project break-even.
  - When NPV is zero, value is neither created nor destroyed.
- It is the actual rate of return earned from the investment.
  - It answers the question – "what is the return on the investment?"
- Decision Rule –

If IRR > required rate of return, we should accept the project.

NPV Payoff Profile

discount rate	NPV
0.01	55,301.36
0.03	46,368.45
0.05	38,010.30
0.07	30,178.53
0.09	22,829.67
0.11	15,924.64
0.13	9,428.20
0.15	3,308.51
0.17	-2,463.25
0.19	-7,913.20
0.21	-13,065.07
0.23	-17,940.46
0.25	-22,559.04
0.27	-26,938.78
0.29	-31,096.12
0.31	-35,046.08
0.33	-38,802.47





NPV and IRR - Excel

- Spreadsheet (NPV_IRR)
- Using the NPV function
  - The first component is the required return, the second component is the range of cash flows *beginning with* ***year 1***
  - Subtract the initial investment at year 0 after computing the PV

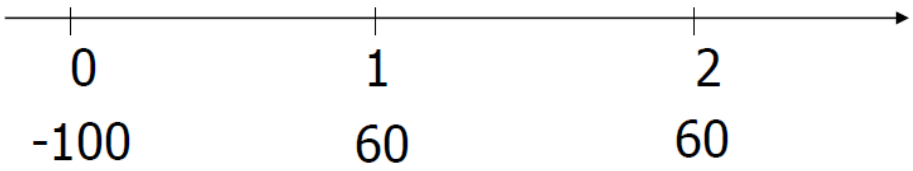
```
=NPV(0.12,63120,70800,91080)-165000
```

- Using the IRR function

```
=IRR({-165000,63120,70800,91080})
```

You are looking at a new project and have estimated the following cash flows based on net income and book value data:			
Investment at Year 0	CF = -165,000		
Year 1:	CF = 63,120		
Year 2:	CF = 70,800		
Year 3:	CF = 91,080		
Your required return for assets of this risk is 12%			

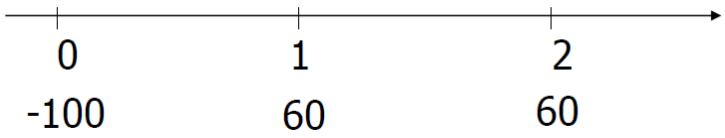
IRR (NO – EXAM)

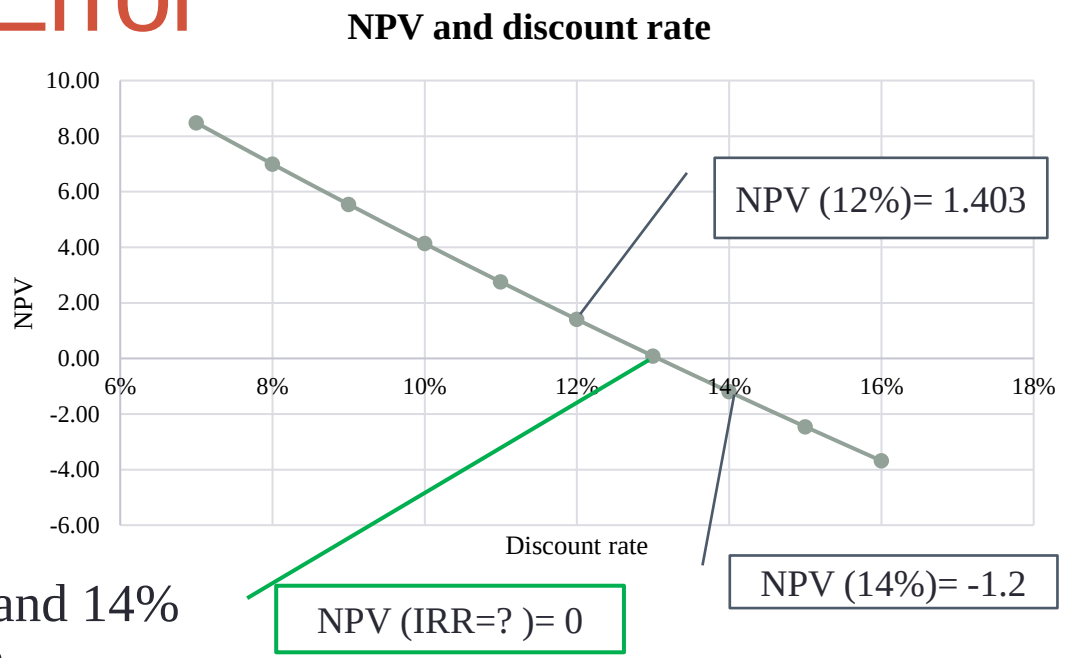


$$-100 + \frac{60}{1+IRR} + \frac{60}{(1+IRR)^2} = 0, \text{ set: } \frac{1}{1+IRR} = x$$

$$\mathrm{x}(IRR) = 0.1306 \ \ or \ \ 13.06\%$$

IRR – Trial and Error





Trial and Error

1) Compute the NPV when *r* is 12% and 14%
2) Use **interpolation** to compute IRR

NPV (r=12%) =1.403 NPV (r=14%) = -1.2

$$\frac{IRR-12\%}{14\%-12\%}=\frac{0-1.403}{-1.2-1.403} \quad \text{or} \quad \frac{14\%-12\%}{-1.2-1.403}=\frac{IRR-12\%}{0-1.403}$$

Solve for IRR:
IRR=13.08%

If the required rate of return is 15%, we should reject the project.
If the required rate of return is 5%, we can accept the project.

Solve IRR manually

You are looking at a new project and have estimated the following cash flows, net income and book value data:			
Investment at Year 0	CF = -165,000		
Year 1:	CF = 63,120		
Year 2:	CF = 70,800		
Year 3:	CF = 91,080		
Your required return for assets of this risk is 12%			

Questions (15 mins):

- What is the NPV when discount rate is 0.15?
- What is the NPV when discount rate is 0.17?
- What is implied IRR based on Trial and Error around 0.15 and 0.17?

IRR – Example

In this example, the NPV rule and the IRR rule lead to ***identical*** accept-reject decisions. Assume required return = 10%

IRR =19.44%

If required return < 19.44 then accept

NPV=40.80

NPV is positive and we accept the project as well

Is it always true? **No!**

- **Non-conventional Cash Flow → Multiple IRRs or No IRRs**
- **Mutually Exclusive Projects → Scale or Horizon problems**

IRR & Non-conventional cash flows

Conventional Cash Flow:

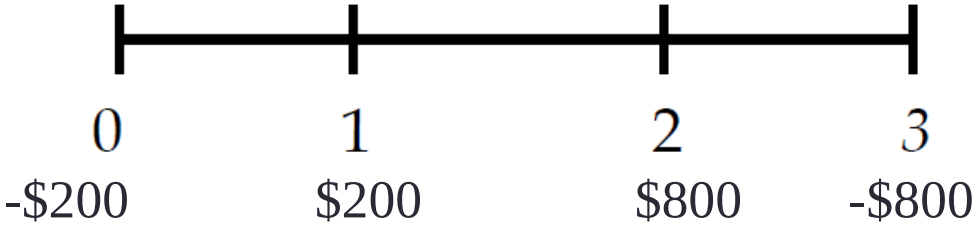
- First cash flow (initial investment) is negative and all the rest are positive.

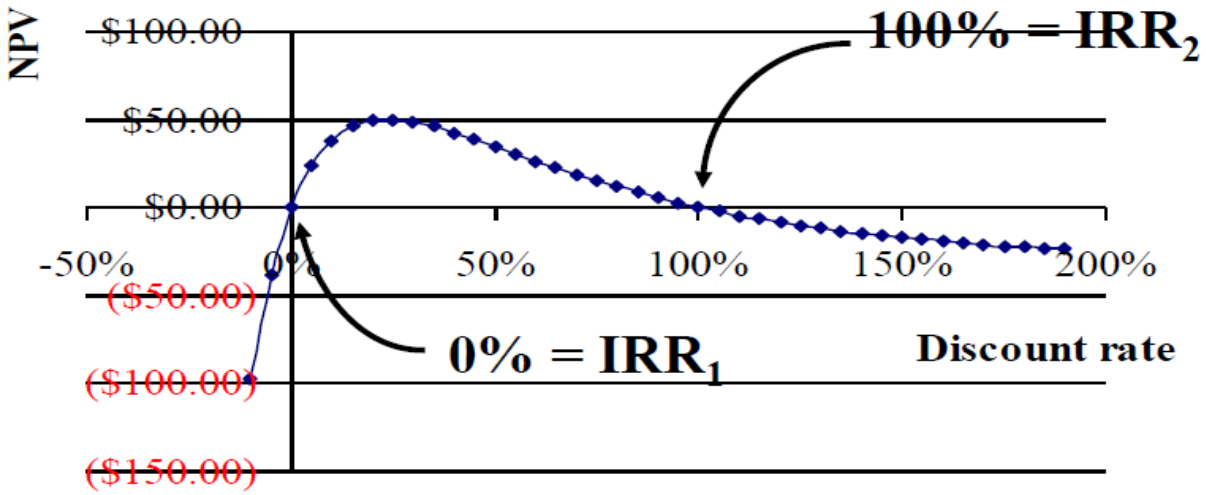
When cash flows are not conventional

- No solutions for IRR
- Multiple IRRs

Multiple IRR & Non-conventional cash flows

Example 1





There are two IRRs for this project

Non-existent IRR & Non-conventional cash flows

Calculate IRR for the following cash flows:

CF1 = -51, CF2 = 100, CF3 = -50

Result: #NUM! – IRR does not exist

IRR & Mutually exclusive projects

Mutually exclusive projects

If you choose one project, you can't choose the other
Example: You own a plot of land, you can either build a petrol station or a house on it, but not both

Intuitively you would use the following decision rules:

NPV – choose the project with the higher NPV
IRR – choose the project with the higher IRR

However, the best project (higher NPV) might not have the highest IRR!

Mutually exclusive projects – Scale Problem

- NPV focuses on the increase in shareholder value, while IRR focuses on the rates of return. NPV may prefer larger-scale projects.

- Scale Problem

  Example: Suppose we can choose one of the following projects (assuming discount rate of 5%). At the end of the project, you can recoup what you invested. Which one should we choose for the shareholders?

  - Project 1 offers 100 with investment of 1,000 ✓
  - Project 2 offers 1 with investment of 1
    - *Project 1: NPV = -1000 + (100 +1000) / $(1+5\%)^1$=48 IRR = 10%*
    - *Project 2: NPV = -1+ (2) / $(1+5\%)^1$= 0.9 IRR=100%*

We are ultimately interested in creating higher value for shareholders not earning higher return on investment! DO NOT use IRR to choose between projects

Mutually exclusive projects – Investment Horizon

Even when projects have the same scale ranking using IRR may be incorrect due to investment horizon**.** NPV may prefer longer-term projects.

- Investment Horizon

You invest 100 at 50% rate of return:

- Short term project (1 year): CF_0 = -100 CF_1 = 150 (100x1.5)
- Long term project (5 year): CF_0 = -100 CF_5 = 759.375 ($100x1.5^5$) ✓

Both projects have IRR = 50%; If the cost of capital is 10% NPV is:

- *Short term project: NPV = 36.36 (NPV = 150/1.1 -100= 36.36))*
- *Long term project: NPV = 371.51 (NPV = 759.375/1.1^5 -100 = 371.51)*

Numerical example of Mutually exclusive projects

Period	Project A	Project B
0	-500	-400
1	325	325
2	325	200
IRR	19.43%	22.17%
NPV	64.05	60.74

Suppose we only have **600** in hand, the required return for both projects is 10%. Which project should you accept and why?

Project A since NPV for A is larger

However, the NPV is affected by the assumption of required rate of return.

Practitioners still want to use IRR.

Incremental IRR

Incremental IRR:

Apply the IRR rule to the difference between cash flows of two mutually exclusive projects.

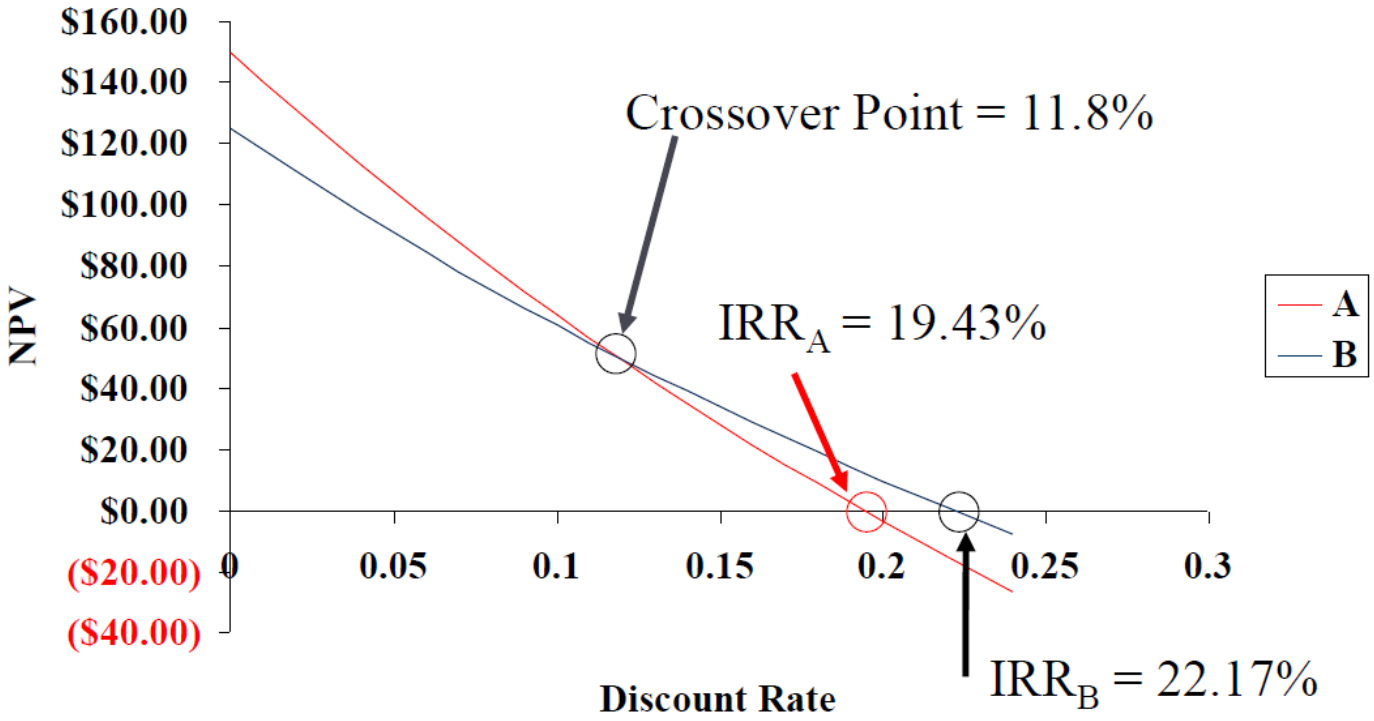
Period	Project A	Project B	***A-B***
0	-500	-400	***-100***
1	325	325	***0***
2	325	200	***125***
IRR	19.43%	22.17%	***11.8%***

INCREMENTAL CASH FLOWS

Excel: IRR ({-100,0,125})

INCREMENTAL IRR, cross-over rate

Incremental IRR



IRR for B is always higher than IRR for A, however, depending on the required return

- **If required return < crossover point → Project A**
- **If required return > crossover point → Project B**

If the required return is less than the crossover point of 11.8%, there is a ranking conflict between NPV and IRR

Incremental IRR

Cross-over rate (Incremental IRR): 11.8% (A-B)
Incremental IRR tells us the discount rate at which project A and project B are equivalent.

Period	Project A	Project B
0	-500	-400
1	325	325
2	325	200
Sum of cash flows	150=-500+325+325	125=-400+325+200

- If required rate (discount rate) < 11.8% - Project A since Project A has larger sum of value *assuming zero discount rate*, meaning that A fares better with lower discount rate
- If discount rate > 11.8% - Project B

Example 2 (show excel formula)

Consider the following two mutually exclusive projects

Year	Cash flow (A)	Cash flow (B)
0	-252000	-24000
1	18,000	14,400
2	36,000	12,600
3	38,400	11,400
4	510,000	9,800

What is the Excel formula for cross-over rate?

IRR ({_____})

Which project would you go for if the discount rate is less than the cross – over rate?

Solutions

Consider the following two mutually exclusive projects			
Year	Cash flow (A)	Cash flow (B)	**A-B**
0	-252000	-24000	**-228000**
1	18,000	14,400	**3,600**
2	36,000	12,600	**23,400**
3	38,400	11,400	**27,000**
4	510,000	9,800	**500,200**
Sum of cash flows	350400	24200	
NPV@15%	$107,716.12	$11,148.02	
IRR	26.90%	38.27%	**26.23%**
What is the cross-over rate?	**26.23%**		
Which project would you go for if the discount rate is 15%?	Since 15% is larger than cross-over rate, Project A fares better in low discount rate environment, go for project A	Which project would you go for if the discount rate is 30%?	Since 30% is larger than cross-over rate, go for project B

=IRR({-228000,3600,23400,27000,500200})

If the discount rate is less than cross-over rate, go for project A (sum of CFs for A is higher)

Past – Exam question

Consider the following two mutually exclusive projects

What is the Excel formula for the calculating the incremental IRR/cross-over rate that equates the value Project A to the value of Project B.

A. IRR(−25000,800,1600,5500)

B. IRR(−25000,800,1600,5500) −IRR(−23500,3000,2000,500)

C. IRR(−1500,2200,400, −5000)

D. IRR(1500,2200,400, −5000)

	Project A	Project B
Year 0	−25000	−23500
Year 1	800	3000
Year 2	1600	2000
Year 3	5500	500

IRR – Value Additivity

Period	Project A	Project B
0	-500	-400
1	325	325
2	325	200
IRR	19.43%	22.17%
NPV	64.05	60.74

NPV_A=64.05, NPV_B=60.72, if you are allowed to invest in both,

- $NPV_{A+B} = NPV_A + NPV_B = 124.77$

IRR_A=64.05, IRR_B=60.72, if you are allowed to invest in both

- $IRR_{A+B} \neq IRR_A + IRR_B \neq 41.6\%$
- Proper way: Add the cash flows for both project A and project B
- IRR({-900, 650, 525 }) = 21%

NPV & IRR Conflicts – Summary

In theory: whenever there is a conflict between NPV and another decision rule, you should ***always*** use NPV since IRR is unreliable in the following situations

- Non-conventional cash flows
  - Multiple IRR
  - Non-existent IRR
- Mutually exclusive projects
  - Differences in investment scales (larger vs. smaller projects)
  - Differences in investment horizons (long- vs. short-term projects)

In Practice: IRR may have a practical advantage over the NPV

- For stand-alone (independent) projects with conventional cash flows, IRR and NPV are interchangeable techniques.
- We can't estimate the NPV unless we know the appropriate discount rate, but we can still estimate the IRR (intuitively appealing)

Exercise (20 mins)

Q1. Consider following two mutually exclusive projects.

1) What are the NPV for both projects with 8% required rate of return? What is the Excel formula for computing the IRR for the projects?

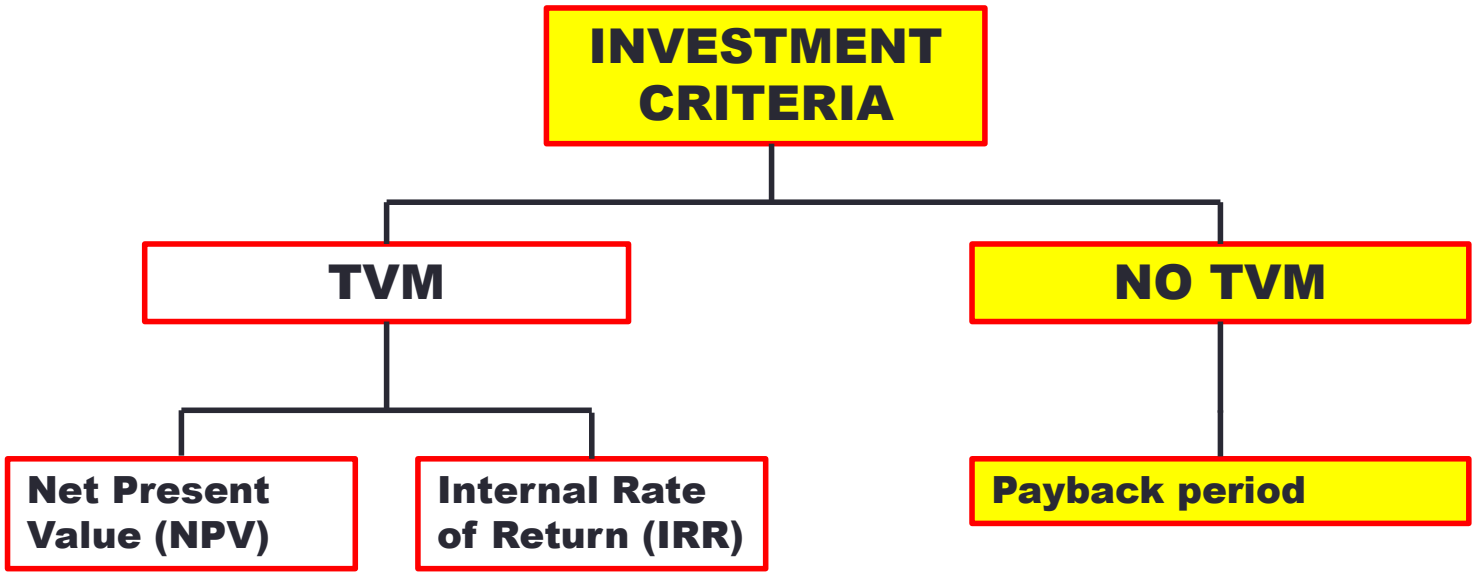
2) What is the Excel formula for computing the cross-over rate for the project?

3) Which project would you choose if the cross-over rate is 5.89% and the required rate of return is 8%?

Year	Project A	Project B
0	-21,000	-10,500
1	4,000	2,500
2	7,500	3,800
3	8,200	3,600
4	6,300	3,900

Q2. An investment provides you with **four** annual cash flows of 400 **starting in year 3.** If you require 5% return on your funds what is the present value of these cash flows? What is the net present value of the investment if the project requires an initial cost of 1000, is it worth investing?

Lecture outline



Payback period

- How long does it take to get the initial cost back?
- Computation:
  - Accumulate the future cash flows
  - Stop until the initial investment has been recovered
  - Compute the fraction of years
- Decision Rule – ***Accept if the payback period is less than some pre-set maximum payback***

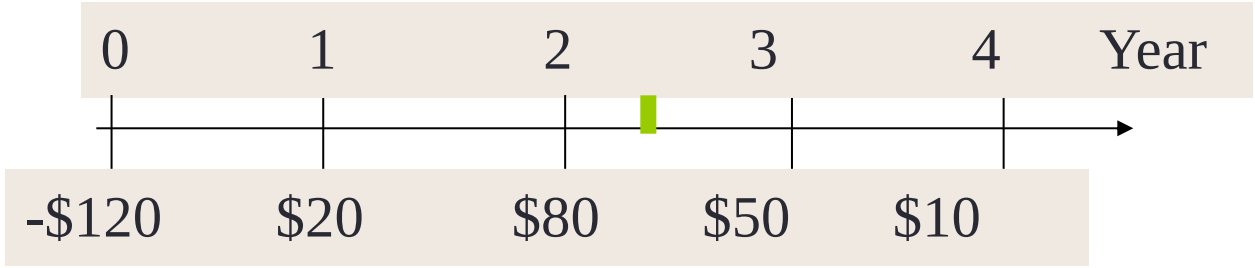
Payback period

Year	Project A	Project B
0	– 1000	– 1000
1	500	400
2	500	400
3	0	400
4	0	400
Payback	2 years	2.5 years

If maximum payback < 2.3 years – Accept A

Payback period – Example 1

- What is the payback period for the following project?



First Year $20 (paid back) → −120+20 = − $100
Second Year $80 (paid back) → −$100+80 = −$20
Third Year $50 (paid back) → −$20+50 = $30

The payback period should be between 2 and 3 years.
The payback period = 2 + 20/50 =2.4 years.

Payback period – Practice

- What is the payback period for the following project?

Year	Long term project	Short term project
0	-250	-250
1	100	100
2	100	200
3	100	0
4	100	0

Payback period – Practice

- What is the payback period for the following project?

Year	Long term project	Short term project
0	-250	-250
1	100	100
2	100	200
3	100	0
4	100	0

Long-term = 2+50/100= 2.5 years
Short-term = 1+150/200=1.75 years

Payback period

- **Advantages**

  Easy to understand
  Adjusts for uncertainty in later cash flows

- **Disadvantages**

  Ignores the time value of money
  Requires an arbitrary cut-off point and ignores cash flows beyond the cut-off date
  Biased against long term project, such as R&D and new projects

Project Selection

- Evaluation of Multiple Projects with Different Resource Requirements/Constraints
- Profitability Index
- Shortcomings of the Profitability Index

Profitability Index - Formula

$$\text{Profitability Index} = \frac{\text{Value Created}}{\text{Resource Consumed}} = \frac{\text{NPV}}{\text{Resource Consumed}}$$

Profitability index tells us $ return for every $1 invested in the project, or to be more precise:

Value created in terms of NPV per unit of initial investment.

PI- Human Resource Constraint

Problem

Your division at NetIt, a large networking company, has put together a project proposal to develop a new home networking router. The expected NPV of the project is $17.7 million, and the project will require 50 software engineers. NetIt has a total of 190 engineers available, and the router project must compete with the following other projects for these engineers:

Project	NPV ($ millions)	Engineering Headcount
Router	17.7	50
Project A	22.7	47
Project B	8.1	44
Project C	14.0	40
Project D	11.5	61
Project E	20.6	58
Project F	12.9	32
Total	**107.5**	**332**

How should NetIt prioritize these projects?

PI- Human Resource Constraint

Solution

The goal is to maximize the total NPV we can create with 190 employees (at most). We compute the profitability index for each project, using Engineering Headcount in the denominator, and then sort projects based on the index:

Project	NPV ($ millions)	Engineering Headcount (EHC)	Profitability Index (NPV per EHC)	Total EHC Required
Project A	22.7	47	0.483	47
Project F	12.9	32	0.403	79
Project E	20.6	58	0.355	137
Router	17.7	50	0.354	187
Project C	14.0	40	0.350	
Project D	11.5	61	0.189	
Project B	8.1	44	0.184	

We now assign the resource to the projects in descending order according to the profitability index. The final column shows the cumulative use of the resource as each project is taken on until the resource is used up. To maximize NPV within the constraint of 190 employees, NetIt should choose the first four projects on the list. The resource constraint forces NetIt to forgo three otherwise valuable projects.

Profitability index - Issues

- **There can only be a single relevant resource constraint**

  What if there is more resource constraints? Example: Capital limit and Headcount constraint.

- **The set of projects completely exhausts the available resource**

  Previous example: What if there was a project requiring 3 employees but with NPV = 120,000?

  Profitability index = 0.12/3 = 0.04 – Based on PI the worst project!, but we can still select it.

PI- Financial Constraint (1/2)

ABC Corp. has a total of 1000$ available to spend and has to choose from the following projects. Which projects would it go for?

Project	NPV	Initial investment	PI = NPV / Initial Investment
A	240	100	2.4
B	500	380	1.32
C	1200	800	1.5
D	50	10	5
E	120	100	1.2

Ranking projects in a descending order based on PI,
We can go for project D, A, and C, consuming 910 dollars

PI- Financial Constraint (2/2)

ABC Corp. has a total of 1000$ available to spend and has to choose from the following projects. Which projects would it go for?

Project	NPV	Initial investment	PI = NPV / Initial Investment
A	240	100	2.4
B	500	380	1.32
C	1200	800	1.5
D	50	10	5
E	**5**	**5**	**1.0**

Ranking projects in a descending order based on PI, we can go for project D, A, and C, consuming 910 $.

We still have space have Project E (costing 5$) even if E has the worst PI

In the end, we select projects D, A, C and E.

Exercise Q3 – Q5

3. Profitability Index: The Sisyphean Company is planning on investing in a new project. This will involve the purchase of some new machinery costing $450,000. The Sisyphean Company expects cash inflows from this project as detailed below

Year One	Year Two	Year Three	Year Four
$200,000	$225,000	$275,000	$200,000

The appropriate discount rate for this project is 16%. What is the profitability index for this project?

4 (homework) Project Ranking: ABC Corp. has a total of 800$ available to spend and has to choose from the following projects. Which projects would it go for?

Project	NPV	Initial investment
A	240	50
B	500	350
C	1200	700
D	50	40
E	120	100

5 (homework) Consider the following two mutually exclusive projects. Whichever project you choose based on the following criteria, if you will require a 15% return on your investment.

1) If you apply the payback criterion, which project will you choose?
2) If you apply the NPV criterion, which project will you choose?
3) What is the NPV for Project A when the discount rate is 28%, what its NPV when the discount rate is 26%. And then use **Trial and Error** to estimate its IRR for project A.

Year	Cash flow (A)	Cash flow (B)
0	-252,000	-24,000
1	18,000	14,400
2	36,000	12,600
3	38,400	11,400
4	510,000	9,800