INTEREST RATE

Lecture Outline

- Exchange between APR (quoted rate) and EAR (effective annual rate)
- Outline mortgage payment schedules
- The relationship between nominal interest rate, inflation rate, real interest rate
- Understand the definition of Yield Curve

Interest rate

- Sometimes the way a rate is quoted is the result of tradition or legislation
- Interest rates:
  - **Quoted rate**: most available rate
  - **Effective interest rate**: most informative rate

Annual Percentage Rate *(APR)*

- APR **(Quoted rate)** = The interest charged per period times the number of periods per year
  - Interest rate charged per period = APR / number of periods per year
  - Normally, the APR is the quoted interest rate in financial news or newspapers.

Compounding APR

The same quoted APR can correspond to many different compounding frequencies per year:

- 10% compounded semi-annually: 10%/2 = 5% every 6 months
- 10% compounded quarterly: 10%/4 = 2.5% every 3 months
- 10% compounded monthly: 10%/12 = 0.83% every month
- 10% compounded daily: 10%/365 = 0.02739% every day

➢ **They are NOT the same rates**

Example: 10% compounded semi-annually

A saving account offers a 10% annual rate of return, but pay interests every 6 months.

Semi-annual rate: 10% / 2 = 5%

Assume you have 1000 dollars saved in the account.

In 6 months: 1000 x (1 + 5%) = 1050

In 12 months: 1050 x (1 + 5%) = 1102.5

Example: 10% compounded quarterly

A saving account offers a 10% annual rate of return, but pay interests every 3 months/every quarter.

Quanterly rate: 10% / 4 = 2.5%

Assume you have 1000 dollars saved in the account.

In 3 months: 1000 x (1 + 2.5%) = 1025

In 6 months: 1025 x (1 + 2.5%) = 1050.625

In 9 months: 1050.625 x (1 + 2.5%) = 1076.89

In 12 months: 1076.89 x (1 + 2.5%) = 1103.82

Compounding APR

- **The same quoted APR can correspond to many different compounding frequencies per year:**
  - 10% compounded semi-annually: 10%/2 = 5% every 6 months
  - 10% compounded quarterly: 10%/4= 2.5% every 3 months
  - 10% compounded monthly: 10%/12 = 0.83% every month
  - 10% compounded daily: 10%/365= 0.02739% every day

For 1000 dollars savings at the start of the year, how much do you yield at the end of the year?

- Annually:
  - $1 \times (1+0.10) = 1.10$
  - 1100 if interests are paid (once a year) each year.
- Semi-annually:
  - $1 \times (1 + 0.05)^2 = 1.1025$
  - 1102.5 if interest are paid each 6 month.
- Quarterly:
  - $1 \times (1 + 0.025)^4 = 1.1038$
  - 1103.8 if interests are paid each quarter.
- Monthly:
  - $1 \times (1 + 0.0083)^{12} = 1.1042$
  - 1104.2 if interests are paid each month.
- Daily:
  - $1 \times (1 + 0.0002739)^{365} = 1.1051$
  - 1105.1 if interests are paid each day.

Effective Annual Rate (EAR)

- **Definition**: the actual rate paid (or received) after accounting for compounding that occurs during the year
- Used to compare two alternative investments with different compounding periods

$$EAR = \left[1 + \frac{\text{Quoted Rate}}{m}\right]^{m} - 1$$

m: the number of times interest is compounded per year

Computing EAR - Example

You are looking at two savings accounts.

One pays 5.25%, with daily compounding.

The other pays 5.3% with semi-annually compounding.

Which account should you use?

Think about it for 5 minutes.

How to use EAR and APR

- A bank is offering 12% (APR, quoted rate) compounded quarterly. You put $100 in an account

  What is the EAR?

  $=(1+12\%/4)^4-1=12.55\%$

- How much will you have at the end of 2 years?

  - ***Method 1 (8 quarters)***: $\$100\times(1 + 0.12/4)^8 = \$126.68$
  - ***Method 2 (2 years)***: $\$100\times(1 + 0.1255)^2 = \$126.68$

Loan Type

- There are unlimited possibilities of how loan principal and interest is paid

- Three basic types of loan:

  - Pure discount loans
  - Interest-only loans
  - Amortised loans:
    - *Fixed Principal*
    - *Fixed Payments*

Pure Discount Loan

The borrower receives money today and repays a single lump sum at the end of the loan (without any (other) interest payments).

- Examples:
  - U.S. Government Treasury bills (The principal amount is repaid at some future date, without any periodic interest payments)

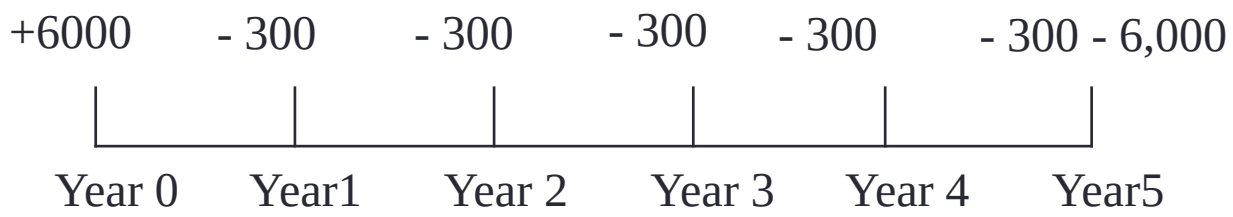
Suppose your firm borrows $6,000 today from a bank and will pay back $10,000 in five years. What is the interest (rate) cost of the loan?

$$\frac{10{,}000}{(1+r)^5} = 6000 \qquad r = 10.76\% \qquad r = \left(\frac{10000}{6000}\right)^{1/5} - 1 = 10.76\%$$

Interest-only Loans

Interest-only loans require payment of *interest* each period and the repayment of the principal at a later date

- Examples:
  - U.S. Government Treasury bonds (The principal amount is repaid at some future date, with periodic interest payments)



Amortized Loans

Amortized loans

- Principal is repaid over time.
  - The amount of principal is decreasing over time, that is, the principal will be progressively reduced to zero.
  - As a part of the principal gets repaid, interest is calculated only on the reminder of the principal
  - Periodic payments = interest + repayment of a portion of the principal.

Example 1

An Example: A company borrow $4,000 4-year loan at 8% from a bank. What is the amortization schedule for:

Fixed principal: Principal is constant each year, which is $4000/4=$1000

- Periodic payment = $1,000 + interest

Fixed payment: The periodic payment is constant each year, and is computed by using Annuity formula

Fixed Principal

Year	Beg. Balance	Periodic Payment	Interest Paid	Principal Paid	End. Balance
1	4,000	1,320	320	1,000	3,000
2	3,000	1,240	240	1,000	2,000
3	2,000	1,160	160	1,000	1,000
4	1,000	1,080	80	1,000	0
Total		4,800	800	4,000	

Fixed payment (1/2)

Calculate annuity:

$$PV = \frac{C}{r} \times \left(1 - \frac{1}{(1+r)^T}\right)$$

$$4000 = \frac{C}{8\%} \times \left(1 - \frac{1}{(1+8\%)^4}\right)$$

$$C = \frac{4000\ x\ 8\%}{1 - \frac{1}{(1+8\%)^4}}$$

$$C = 1{,}207.68$$

Fixed payment (2/2)

C=1,207.68

Year	Beg. Balance	Periodic Payment	Interest Paid	Principal Paid	End. Balance
1	4,000.00	1,207.68	320.00	887.68	3,112.32
2	3,112.32	1,207.68	248.99	958.70	2,153.62
3	2,153.62	1,207.68	172.29	1035.39	1,118.23
4	1,118.23	1,207.68	89.46	1,118.23	0.00
Total		4,830.73	830.73	4,000.00	

Example 2: a 2-year loan

A company borrow $4,000 2-year loan at 8% from a bank. What is the amortization schedule for:

Fixed principal:

Year					
1	4,000				
2					

Fixed payment:

Year					
1	4,000				
2					

Continuously Compounded Rate

Recall that EAR for different frequencies of compounding (APR=12%) is

Quarterly	$(1+12\%/4)^4-1=12.5509\%$
Monthly	$(1+12\%/12)^{12}-1=12.6825\%$
Daily	$(1+12\%/365)^{365}-1=12.7475\%$
Continuously	$\mathbf{e^{0.12} - 1 = 12.7497\%}$

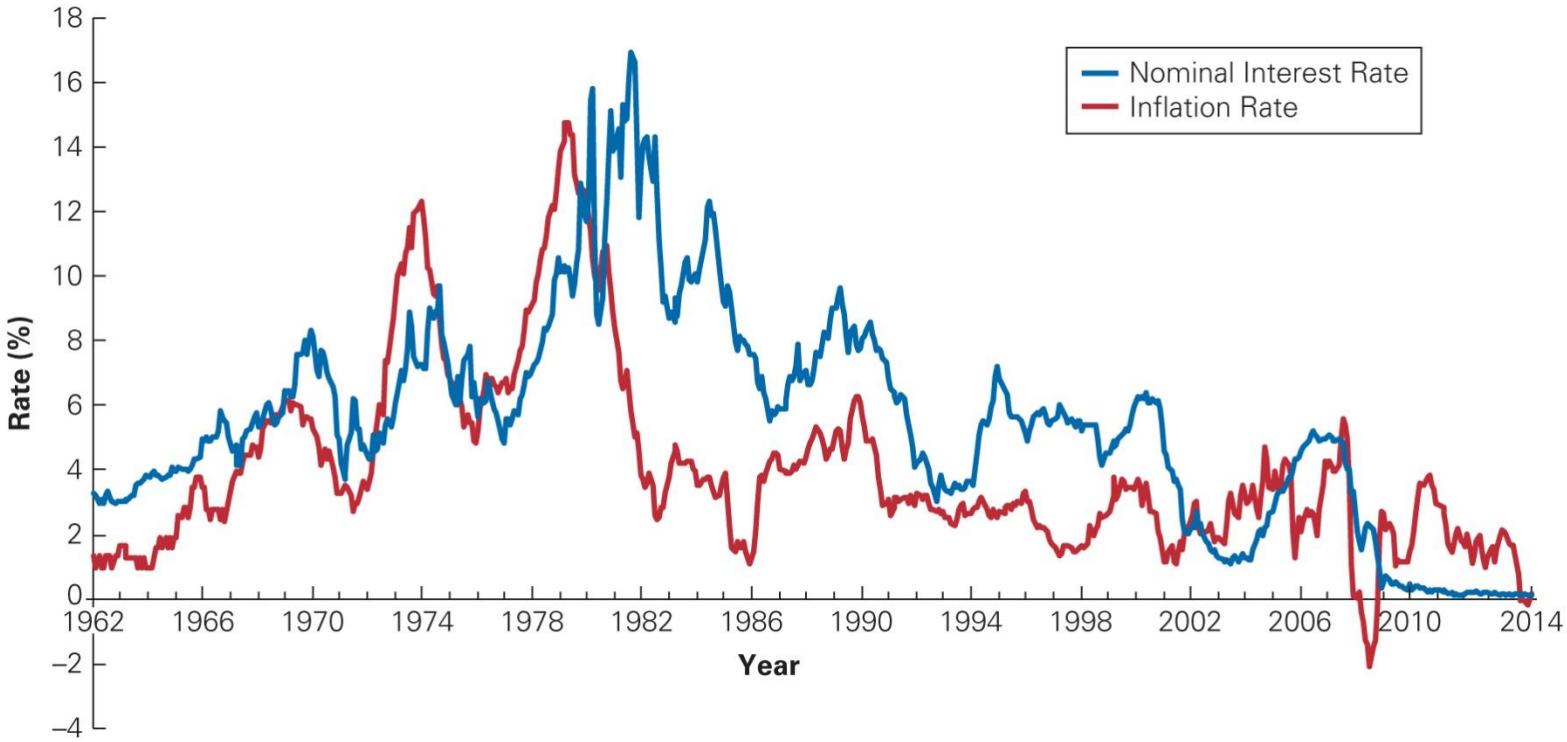
- The EAR for a Continuously Compounded APR

$$(1 + EAR) = e^{APR}$$

- The Continuously Compounded APR for an EAR

$$APR = \ln(1 + EAR)$$

Interest Rate and Inflation



Interest rates are average three-month Treasury bill rates and inflation rates are based on annual increases in the U.S. Bureau of Labor Statistics' consumer price index. Note that interest rates tend to be high when inflation is high.

Nominal Rates

- Nominal rates: Interest rate of returns that have not been adjusted for inflation.
- Real rates: Interest rates of returns that have been adjusted for inflation.
  - The percentage change in terms of purchasing power.

Example

Suppose we have $1,000, and a beer costs $2.00.

- We can buy 500 beers today
- If we invest the $1,000 and it grows to $1,100 for a rate of 10% in one year.
- Normal rate measures the percentage change [(ending value – beginning value)/ beginning value] in money.

Nominal Rate: ($1,100-$1,000)/$1,000 = 10%

Now suppose the rate of inflation is 5%, so that the price rises to $2.10 in one year

- We can buy 1,100/2.1 = 523.811 beers in one year.
- Real rate measures the percentage change [(ending value – beginning value)/ beginning value] in purchase power.

Return in beers (Real Rate) : (523.81 -500) /500 = 4.76%

Real Rates and Inflation

Real Rate of Return = $\frac{1+Nominal\ rate}{1+Inflation\ rate} - 1$

≈ nominal – inflation

- In the previous example, the normal rate is 10%, the inflation rate is 5%, what is the real rate of return?

$$\frac{1+10\%}{1+5\%} - 1 = 0.0476$$

Term Structure of Interest Rate

Yield Curve

- A plot of interest rate (yield of a government bond) against the time to maturity is known as a *yield curve.*

Yield (Bund)

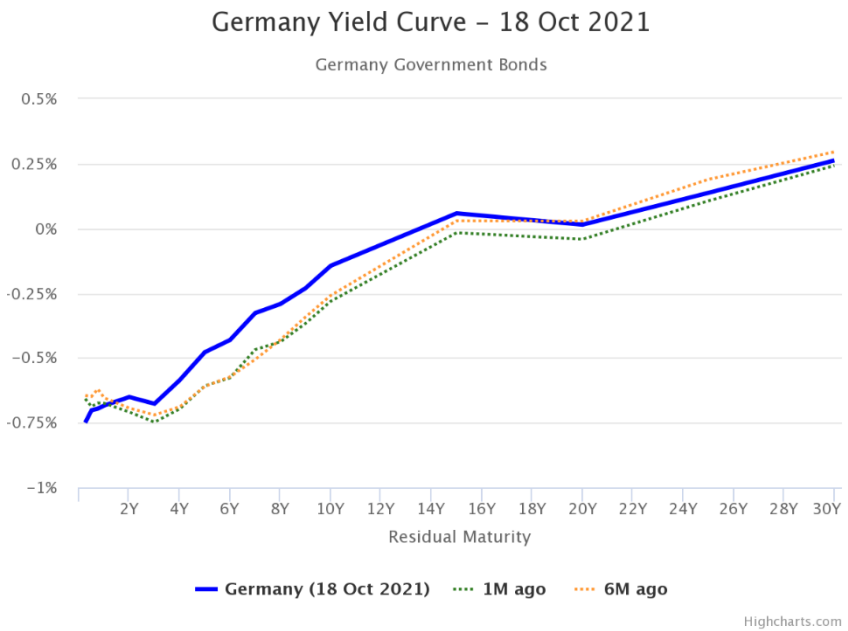
Residual Maturity	Yield			
	Last	Chg 1M	Chg 6M	Last
3 months	-0.750%	-9.2 bp	-10.5 bp	
6 months	-0.702%	-1.4 bp	-5.3 bp	
9 months	-0.696%	-2.2 bp	-7.6 bp	
1 year	-0.684%	-0.9 bp	-3.0 bp	100.69
2 years	-0.651%	+5.8 bp	+4.3 bp	101.31
3 years	-0.677%	+7.1 bp	+4.3 bp	102.06
4 years	-0.586%	+11.1 bp	+10.3 bp	102.38
5 years	-0.478%	+13.0 bp	+13.2 bp	102.42
6 years	-0.431%	+14.6 bp	+14.2 bp	102.63
7 years	-0.327%	+14.1 bp	+17.9 bp	102.32
8 years	-0.292%	+14.6 bp	+13.8 bp	102.37
9 years	-0.231%	+13.6 bp	+11.1 bp	102.10
10 years	-0.145%	+13.8 bp	+11.5 bp	101.46
15 years	0.058%	+7.5 bp	+2.9 bp	99.13
20 years	0.014%	+5.6 bp	-1.4 bp	99.72
25 years	0.137%	+3.1 bp	-5.2 bp	96.64
30 years	0.262%	+2.0 bp	-3.3 bp	92.45

Source: https://www.worldgovernmentbonds.com/country/germany/
[accessed date: Sep, 2021]

A 2-year bund:You can pay 101.31 euros to buy a 2-year German Bund (lending a pure discount loan to German government). In two years, German government will repay you 100 euros (face value).
A 15-year bund: You can pay 99.13 euros to buy a 15-year German Bund. In 15 years, German government will repay you 100 euros (face value).

Questions:
What is the interest rate/yield for this 2-year bund?
What is the interest rate/yield for a 15-year bund?
Why is the positive (for example, +5.8bp for Chg 1M at 2 years bond) change in red color (red means market is trending down)?
Is the yield for the long-term bond higher or lower?

Germany Yield Curve

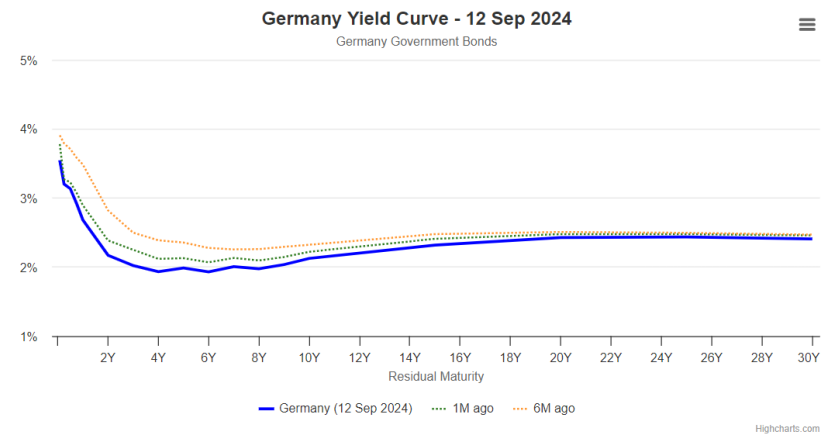


Source: https://www.worldgovernmentbonds.com/country/germany/
[accessed date: **Oct, 2021**]

- **Conventional: Upward-Sloping curve**
  - Long-term bond have higher yield than short-term bond since long-term bond is perceived to be more uncertain/risker.

Germany Yield Curve

Germany Yield Curve



Source: https://www.worldgovernmentbonds.com/country/germany/
[accessed date: **Sep, 2024**]

- **Downward-Sloping curve**
  - Long-term bond have lower yield than short-term bond

REUTERS

German yield curve close to least inverted level since March ahead of U.S. data

Stefano Rebaudo
October 5, 2023 • 3 min read

By Stefano Rebaudo

Oct 5 (Reuters) - Euro zone borrowing costs were mixed on Thursday, after a bond selloff paused the day before as yields hit crucial levels and the German curve reached its least inverted since March.

This week's main economic focus will be Friday's jobs report for September -- which is expected to show that employers added 170,000 jobs -- but investors will closely watch the weekly jobless claims, due later in the session.

Robust economic data coupled with central bank officials' remarks from both sides of the Atlantic claiming that rates will stay at high levels for an extended period triggered a bond selloff mainly on the long end of the curve.

Bond prices move inversely to yields.

The gap between Germany's 2-year and 10-year government bond yields was at -23 bps after hitting -20.9 the day before, its highest since March 20.

An inverted curve, usually a reliable indicator of a future recession, means markets are pricing events that would trigger central bank rate cuts.

Weak economic data might boost expectations that central banks could cut rates soon keeping the yield curve inverted.

On Wednesday, U.S. jobs data helped give some respite to the bond selloff, which affected mostly the long end of the curve.

The ADP National Employment Report showed that U.S. private payrolls increased far less than expected in September.

Germany's 10-year Bund yield was down 0.5 basis points (bps) at 2.93% on Thursday after hitting 3.024% for the first time since July 2011 the day

- News about German yield curve

Recession and Yield Curve

- **Inverted yield curve:**
  - Downward-Sloping curve
  - Long-term bond have lower yield than short-term bond
  - Bond investors expect for a decline in longer-term interest rates (expect the Fed to reduce rate to stimulate economy in the future)
- **Inverted yield curve is a harbinger of the prospects for recession**

Exercise (30 minutes)

Q1. How much are you effectively earning with an APR (annual percentage rate) of 20% with quarterly compounding?
Q2. You borrow 5,000 3-year loan at 5% from a bank. You are using the amortization schedule of Fixed Payment Schedule, what is your amortization scheme? List your interest and principal payments every year.
Q3. You borrow 5,000 3-year loan at 5% from a bank. You are using the amortization schedule of Fixed Principal Schedule, what is your interest payment at the end of second year?
Q4. How much are you effectively earning with an APR (annual percentage rate) of 20% with continuously compounding?

Q5. An investment offers a 15% total return over the coming year. You think the total real return on this investment will be only 9%. What do you believe the inflation rate will be over the next year?